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Effect of looking backward on traffic flow in a cooperative driving car following model

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Abstract. An extended car following model is proposed by incorporating intelligent transportation system and the backward looking effect under certain condition in traffic flow. The neutral stability condition of this model is obtained by using the linear stability theory. The results show that anticipating the behavior of vehicles preceding and following one vehicle could lead to appreciable stabilization of traffic system. From the simulation of space-time evolution of the vehicle headways, it is shown that the traffic jam could be suppressed efficiently via taking into account the information about the motion of two preceding vehicles and one following vehicle, and the analytical result is consistent with the simulation one.

PACS. 64.60.Fr Equilibrium properties near critical points, critical exponents – 05.70.Fh Phase transitions: general studies – 05.70.Jk Critical point phenomena

1 Introduction

Traffic jam is an important issue from the viewpoint of transportation efficiency and reduction in pollution, which has thus attracted much attention recently. As is well-known, if the traffic flow is in the congested state, much carbon dioxide and noise will be generated and plenty of energy will be wasted. So it is necessary to raise the transportation efficiency and prevent traffic jams. In recent years, automatic driving control systems have been utilized as a part of intelligent transport system (for short, ITS). One of the targets of ITS is to suppress the appearance of the traffic jam.

A number of researchers plunged into the investigation. Helbing [1] presented an improved gas-kinetic traffic model, which differs from others mainly by introducing non-local interaction term that takes into account the space requirements of vehicles and the correlations of successive vehicle velocities. The model reflected the anticipation behavior of drivers, which is responsible for a smoothing effect that acts only in the neighboring backward direction. Nagatani [2] put forward an extended optimal velocity model including the vehicle interaction with the next vehicle ahead (i.e., the next-nearest-neighboring interaction). Xue [3] proposed a lattice model of optimized traffic flow with the consideration of the optimal current with the next-nearest-neighboring interaction. After that Ge et al. [4] considered arbitrary number of sites ahead. Lenz et al. [5] constructed a model that a driver can receive the moving information about many vehicles ahead of him/her. In 2004, Hasebe et al. proposed an extended optimal velocity model applied to cooperative driving control system by considering any arbitrary number of vehicles that precede [6], and we called it forward looking optimal velocity model. They found that there exist a certain set of parameters that make traffic flow "most stable" in this model. Ge et al. [7] continued to investigate the dynamic behavior near the critical point of the model.

The above models are related to the forward looking effect, but only few models studied the backward looking effect, such as the models proposed by Nakayama et al. [8] and Hasebe et al. [9]. We think it is reasonable to take the backward looking effect into account.

In this paper, an extended car following model with the consideration of arbitrary number of vehicles ahead and one vehicle following on a single-lane highway is presented. The linear stability theory is given to show the stabilization effect of the new consideration. Numerical simulation is carried out to validate the advantages of the model compared to the other extended car following model.

2 The backward looking models

In 1995, Bando et al. proposed a very charming microscopic car following model called the optimal velocity

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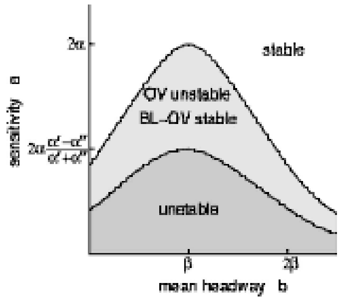

Fig. 1. The phase diagram of the OV model and the BL-OV model.

model (for short, OV model) [10]. It was based on the idea that each vehicle has an optimal velocity, which depends on the distance between it and its preceding vehicle. They used the acceleration equation

$$\frac{d^2x_j(t)}{dt^2} = a\left[V(\Delta x_j(t)) - \frac{dx_j(t)}{dt}\right]. \quad (1)$$

where $x_j(t)$ is the position of vehicle j at time t, $\Delta x_j(t) \equiv x_{j+1}(t) - x_j(t)$ is the headway between car j and car $j+1$ at time t, a is the sensitivity of a driver, and V is the optimal velocity function. Despite its simplicity, the OV model can be employed to describe various properties of real traffic flows, such as the instability of traffic flow, the evolution of traffic congestion, and the formation of stop-and-go waves. Just because the OV model can describe the formation mechanism of traffic jam, many researchers discussed how to stabilize the traffic flow in the context of the extension of the models [5–9].

In the OV model, the appearance of congestion can be suppressed by choosing higher sensitivity. Nakayama et al. [8] considered that a driver could look at the following vehicle as well as the preceding one, which is called the backward looking optimal velocity model (for short, BL-OV model). The BL-OV model consists of the following equation

$$\frac{d^2x_j(t)}{dt^2} = a\left[V_F(\Delta x_j(t)) + V_B(\Delta x_{j-1}(t)) - \frac{dx_j(t)}{dt}\right], \quad (2)$$

where $V_F(\Delta x_j(t))$ is the OV function for forward looking, which is equivalent to the $V(\Delta x_j(t))$ in equation (1). $V_B(\Delta x_{j-1}(t))$ is the OV function for backward looking, which is a function of the headway between the considered vehicle and its following one. The two OV functions have been chosen as

$$V_F(\Delta x_j) = \alpha'[\tanh(\Delta x_j - \beta) + \gamma], \quad (3)$$

$$V_B(\Delta x_{j-1}) = -\alpha''[\tanh(\Delta x_{j-1} - \beta) + \gamma], \quad (4)$$

where α', α'', β and γ are positive constants. The function $V_B(\Delta x_{j-1})$ has the effect of increasing the vehicle velocity, if the headway between two successive vehicles becomes small.

From Figure 1 we can see that the homogeneous flow is stable for both models in the upper region, in the middle region it is unstable only for the OV model, and in the lower region it is unstable for both models. Figure 1 clearly shows that the stable region for the BL-OV model is larger than that for the OV model.

In 2003, Hasebe et al. [9] presented an extended optimal velocity model applied to cooperative driving control system by taking into account arbitrary number of vehicles that precede or follow, which is called hybrid OV model (for short, HB-OV model). The motion equation reads

$$\frac{dx_j^2}{dt^2} = a\Big[V(\Delta x_{j+l_+}, ..., \Delta x_{j+1},\ \Delta x_j,\ \Delta x_{j-1}, ..., \Delta x_{j-l_-}) - \frac{dx_j}{dt}\Big], \quad (5)$$

The model with $l_+ = l_- = 0$ is the original OV model. They concluded that the HB-OV model ($l_+ = l_- = 1$) is a candidate as a dynamical model of cooperative driving system that controls real traffic flow on a highway.

3 An extended OV model

Applying the information provided by ITS, a backward looking anticipation optimal velocity model (for short, BLA-OV model) is presented as follows. The vehicle motion is described by the following differential equation:

$$\frac{dx_j(t+\tau)}{dt} = pV_F(\Delta x_j(t),\ \Delta x_{j+1}(t), ..., \Delta x_{j+n-1}(t)) + (1-p)H(h_c - \Delta x_{j-1}(t))V_B(\Delta x_{j-1}(t)), \quad (6)$$

where n denotes the number of vehicles ahead considered; $\tau = 1/a$ is introduced to denote the delay time with which the vehicle velocity reaches the optimal velocity as the traffic flow is varying; $H(\cdot)$ is the heaviside function. It has been assumed that a driver could obtain the information of n vehicles in front. The vehicle velocity $dx_j(t+\tau)/dt$ is adjusted according to the headways $\Delta x_{j-1}(t)$, $\Delta x_j(t)$, $\Delta x_{j+1}(t), ..., \Delta x_{j+n-1}(t)$. Through linear and nonlinear analyses, we concluded that only the information of three vehicles ahead is enough for cooperative driving car following model (i.e., $n = 3$) without including the backward looking effect [7], i.e., the second term on the right side of equation (6). In the BLA-OV model, p stands for the relative roles of the two OV functions, and we set $p = 0.8$ in the later simulation. That is to say, the forward looking effect is more important than the backward looking effect. We think that the preceding vehicles and the following one, which have different effects on the considered vehicle, should be considered separately. So the corresponding OV functions must be different. Besides that, $H(h_c - \Delta x_{j-1}(t))$ as a factor is add to the backward looking term of the dynamical equation (6). Because we think that if the following vehicle is very near to the considered vehicle, the considered one has to accelerate for avoiding collision. So the backward looking effect plays its role only if the headway is less than a certain distance

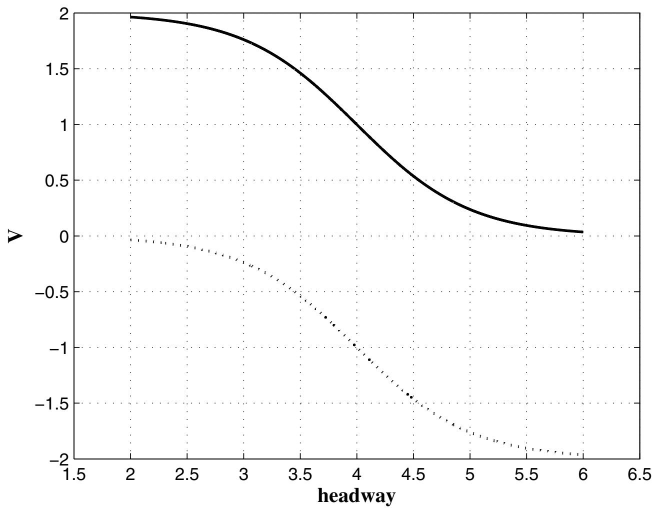

Fig. 2. The dash line and the solid line corresponding to $V_B(\Delta x_{j-1})$ in the BL-OV model and the BLA-OV model respectively, where $\alpha'' = 1,\ \beta = 4.0,\ \gamma = \tanh(4.0)$.

between the successive vehicles, which is set as the safety distance h_c.

We select the following OV functions for the BLA-OV model:

$$V_F(\Delta x_j(t), \Delta x_{j+1}(t), ..., \Delta x_{j+n-1}(t)) = \alpha' \left[\tanh\left(\sum_{l=1}^{n} \xi_l \Delta x_{j+l-1}(t) - \beta\right) + \gamma\right], \quad (7)$$

$$V_B(\Delta x_{j-1}) = \alpha''[-\tanh(\Delta x_{j-1} - \beta) + \gamma], \quad (8)$$

where ξ_l is the weighting function of $\Delta x_{j+l-1}(t)$ in equation (7). It is necessary to point out that ξ_l $(l = 1, 2, ..., n)$ have the following properties:

(1). ξ_l $(l = 1, 2, ..., n)$ decrease monotonically with increasing l, which means $\xi_l/\xi_{l-1} < 1$, for we know that the influence of the vehicles ahead on the vehicle motion reduces gradually as the distance between the considered vehicle and the vehicle ahead increases;

(2). $\sum_{l=1}^{n} \xi_l = 1$, and $\xi_l = 1$ for $n = 1$.

Equation (8) differs from equation (4) as shown in Figure 2. The lower line corresponds to the OV function in the BL-OV model, and the upper line corresponds to that in the BLA-OV model. $V_B(\Delta x_{j-1})$ decreases monotonically with increasing Δx_{j-1}, and it is always positive for only making the considered vehicle accelerate. As the headway Δx_{j-1} is greater than the safety distance h_c, the BLA-OV model is reduced to the cooperative driving OV model [7], and the backward looking effect disappears, which means the second term in the right hand of equation (6) does not work.

In the BLA-OV model, we adopt the first order differential equation with time delay, which is different with the second form of equation (5). While according to the linear stability analysis, the results will not change with the equation forms. Making the Taylor expansion of equation (6) and omitting the higher order term of τ, we could obtain the corresponding second order differential equation. By transforming time derivatives to asymmetric forward differences, equation (6) could be rewritten as the difference equation easily, which is convenient for numerical simulation.

4 Linear stability analysis

The method of linear stability analysis is applied to the extended car following model. It is obvious that the vehicle moves with the constant headway h and the optimal velocity $pV_F(b, b, ..., b) + (1 - p)V_B(b)$ is the steady state solution for equation (6), given as

$$x_j^0(t) = bj + [pV_F(b, b, ..., b) + (1 - p)V_B(b)]t \quad \text{with} \quad b = \frac{L}{N}, \quad (9)$$

where N is the total number of vehicles, and L is the road length.

Suppose $y_j(t)$ to be a small deviation from the steady-state solution $x_j^0(t)$: $x_j(t) = x_j^0(t) + y_j(t)$. Substituting it into equation (6) and linearizing the resulting equation yield

$$\frac{dy_j(t+\tau)}{dt} = pV_F'(b) \sum_{l=1}^{n} \xi_l \Delta y_{j+l-1}(t) + (1 - p)H(h_c - b - \Delta y_{j+l-1})V_B(h + \Delta y_{j-1}) - (1 - p)H(h_c - b)V_B(h), \quad (10)$$

where $\Delta y_j(t) \equiv y_{j+1}(t) - y_j(t)$, and $V'(b) = dV(\Delta x_j)/d\Delta x_j|_{\Delta x_j = b}$. For simplicity, $V_F'(b, b, ..., b)$ is indicated as $V_F'(b)$ in the above equation and hereafter. Here we mainly focus on the condition $b < h_c - |\Delta y_{j-1}|$.

Expanding y_j in the Fourier-modes: $\Delta y_j(t) = \exp(ikj + zt)$, we obtain

$$ze^{z\tau} = pV_F'(b) \sum_{l=1}^{n} \xi_l (e^{ikl} - e^{ik(l-1)}) + (1 - p)V_B'(b)(1 - e^{-ik}), \quad (11)$$

Expanding $z = z_1(ik) + z_2(ik)^2 + ...$, and inserting it into equation (11) lead to the first- and second-order terms of coefficients in the expression of z respectively

$$z_1 = pV_F'(b) + (1 - p)V_B'(b), \quad (12)$$

$$z_2 = -z_1^2\tau - \frac{(1 - p)V_B'(b)}{2} + \frac{pV_F'(b)}{2} \sum_{l=1}^{n} \xi_l (2l - 1), \quad (13)$$

Thus the neutral stability condition is given by

$$\tau = \frac{pV_F'(b) \sum_{l=1}^{n} \xi_l (2l - 1) - (1 - p)V_B'(b)}{2[pV_F'(b) + (1 - p)V_B'(b)]^2}, \quad (14)$$

$$V'_l = \frac{V(b+\Delta y_{j+l_+},...,b+\Delta y_{j+1},\ b+\Delta y_j,\ b+\Delta y_{j-1},...,b+\Delta y_{j-l_-})}{d\Delta y_{j+l}}|_{\Delta y=0},\ -l_- \le l \le l_+,$$

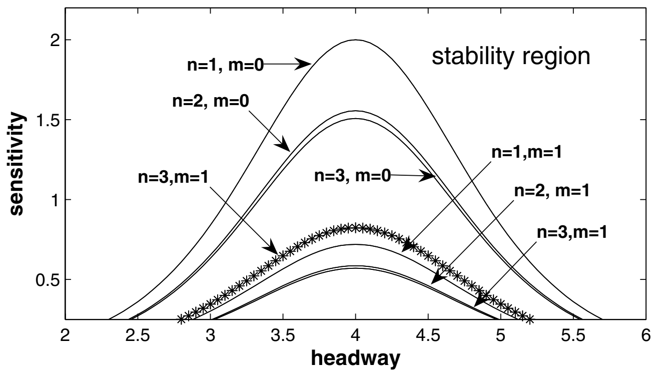

Fig. 3. The phase diagram of the extended OV models, where the upper three lines corresponding to the cooperative driving car following model, the middle line corresponding to the HB-OV model and the lower three lines matching the BLA-OV model respectively, and $n = 3, m = 1$ representing three preceding vehicles and one following vehicle.

Corresponding to the HB-OV model, the neutral stability condition is

$$\tau = \frac{\sum_{l=l_-}^{l_+} V'_l(2l+1)}{2[\sum_{l=l_-}^{l_+} V'_l]^2}. \tag{15}$$

where

(see equation above)

For small disturbances with long wavelengths, the uniform traffic flow is unstable in the BLA-OV model under the condition that

$$\tau > \frac{pV'_F(b)\sum_{l=1}^{n}\xi_l(2l-1)-(1-p)V'_B(b)}{2[pV'_F(b)+(1-p)V'_B(b)]^2}. \tag{16}$$

The neutral stability lines in the parameter space $(\Delta x, a)$ are shown in Figure 3 by the solid line. The upper three lines correspond to the cooperative driving car following model, the middle line corresponds to the HB-OV model and the lower three lines match the BLA-OV model respectively, where n stands for the preceding vehicle and m represents the following vehicle. For the case of $n = 1, m = 0$, the upper neutral stability line is consistent with that of the original OV model. We select the expression of equation (18) for ξ_l, and for the convenience to compare, the parameters in HB-OV model are defined as $V'_{-1} = -1/7,\ V'_0 = 5/7,\ V'_1 = 2/7,\ V'_2 = 1/7$. There exist the critical points (h_c, a_c) for the neutral stability lines such that the uniform state irrespective of vehicle headway is always linearly stable for $a > a_c$, while uniform state in a neighborhood of h_c are unstable for $a < a_c$. The apex of each curve indicates the critical point. The traffic flow is stable above the neutral stability line and traffic jam will not appear. While below the line, traffic flow is unstable. From Figure 3 we can see that:

(i) with taking into account more vehicles ahead, the critical points and the neutral stability curves are lowered, which means the stability of the uniform traffic flow has been strengthened;
(ii) while the backward looking effect is added to the model, the traffic jam can be suppressed efficiently;
(iii) moreover, the backward looking effect plays more important role than the next-nearest-neighboring vehicle;
(iv) the stability region in the BLA-OV model is larger than that in the HB-OV model;
(v) the information of two vehicles ahead is enough for the BLA-OV model.

5 Numerical simulation

For the convenience of simulation, we rewrite equation (6) into difference form.

$$\begin{aligned}\Delta x_j(t+2\tau) - \Delta x_j(t+\tau) = p\tau \Bigg[V_F\left(\sum_{l=1}^{n}\xi_l \Delta x_{j+l}(t)\right)\\ -V_F\left(\sum_{l=1}^{n}\xi_l \Delta x_{j+l-1}(t)\right)\Bigg] + (1-p)\tau\\ \times [H(h_c-\Delta x_j(t))V_B(\Delta x_j(t))\\ - H(h_c-\Delta x_{j-1}(t))V_B(\Delta x_{j-1}(t))].\end{aligned} \tag{17}$$

Computer simulation was carried out for the BLA-OV model described by equation (17). The boundary conditions selected are periodic ones. The initial conditions are chosen as follows: $\Delta x_j(0) = \Delta x_0 = 3.5,\ \Delta x_j(1) = \Delta x_0 = 3.5$ for $j \ne 50, 51,\ \Delta x_j(1) = 3.5–0.5$ for $j = 50$, and $\Delta x_j(1) = 3.5 + 0.5$ for $j = 51$, where the total number of vehicles is $N = 100$ and the safety distance is $h_c = 4.0$. The weighting function in equation (7) is selected tentatively as

$$\xi_l = \begin{cases} 6/7^l & l \ne n \\ 1/7^{n-1} & l = n. \end{cases} \tag{18}$$

The parameters in the OV functions are chosen as $\alpha' = \alpha'' = 1,\ \beta = 4.0,\ \gamma = \tanh(4.0)$.

Figure 4 shows the space-time evolution of the headway for various vehicles in front in the cooperative driving vehicle following model and the BLA-OV model. The patterns (a) and (b) in Figure 4 exhibit the time evolution of the headway profile for the cooperative driving car following model as $n = 1, 2$, and the patterns (c) and (d) correspond to that for the BLA-OV model under the same condition. In patterns (a) and (b), the traffic flow is unstable because the instability condition (16) is satisfied for $n = 1, 2$ in the condition that $p = 1.0,\ a = 1.6$. When small disturbances are added to the uniform traffic flow, they are amplified with time and the uniform

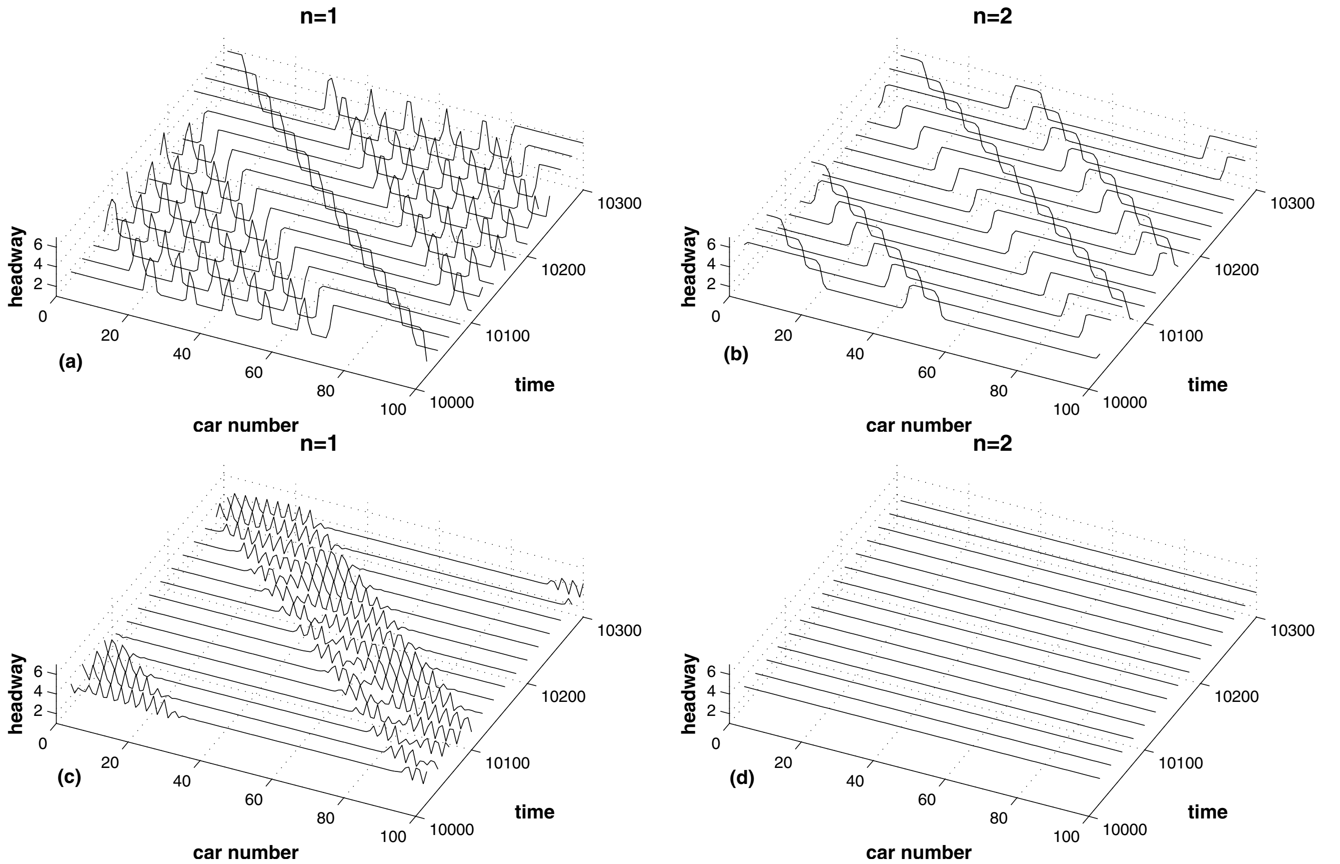

Fig. 4. Space-time evolution of the headway after $t = 10\,000$. The patterns (a), (b) for the cooperative driving car following model, The patterns (c), (d) for the BLA-OV model.

flow changes finally into an inhomogeneous one. In patterns (c) and (d), due to the backward looking effect, the stability is improved greatly for $n = 1, 2$ with the same sensitivity, which demonstrates that the backward looking effect can not be omitted in the OV model. Besides that, only considering the next-nearest-neighboring interaction is enough for suppressing the traffic jam in this situation. With the same sensitivity, as the considered number of vehicles in front increases, the amplitude of the density wave decreases. In pattern (d), traffic flow is uniform over the whole space. Therefore the simulation outcomes are in agreement with analytical results.

6 Summary

We have proposed the BLA-OV model of traffic flow for the purpose of constructing a cooperative driving system. The form of optimal velocity function — equation (7), taking into account the non-local effect, and a novel optimal velocity function — equation (8), describing the backward looking effect are given. Moreover the backward looking effect works only if the headway $\Delta x_{j-1}(x)$ less than the safety distance h_c. The traffic nature has been analytically analyzed by using the linear stability analysis. It has been shown that the combination of backward looking and forward looking effects could further stabilize traffic flow. As $p = 1$, the result is reduced to the cooperative driving car following model. The simulation results confirm the stability analysis for the BLA-OV model and give the optimal state as $n = 2$, that is to say, only the information of two vehicles ahead and one vehicle following is enough for a good cooperative driving.

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